Cherenkov radiation in a photon gas

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Abstract. It is well-known that a charged particle moving with constant velocity in vacuum does not radiate. In a medium the situation can be different. If the so called Cherenkov condition is satisfied, i.e. the particle velocity exceeds the phase speed in the medium, the particle will radiate. We show that a charge moving with a constant velocity in a gas of photons emits Cherenkov radiation, even in the gamma-ray regime, due to nonlinear quantum electrodynamic effects. Our result is evaluated with respect to the radiation background in the early universe, and it is argued that the effect can be significant.

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In 1934, Cherenkov observed the type of radiation now bearing his name [1]. His experimental result was explained by Tamm & Frank [2]. In an isotropic dielectric medium, a charged particle in rectilinear motion satisfying the so called Cherenkov condition, i.e. its velocity exceeds the (parallel) phase speed in the medium in which it moves, will thus radiate [3]. The radiation chock-front, called the Cherenkov cone, is analogous to the Mach cone formed as objects move with supersonic speeds through air. In quantum mechanical terms, the Cherenkov condition corresponds to energy and momentum conservation. Cherenkov radiation has technological uses, e.g. in determining particle velocities.

Quantum electrodynamics (QED) predicts many phenomena with no classical counterparts, such as the Casimir effect and elastic photon–photon scattering [4, 5, 6]. While the former has been experimentally confirmed, the latter has still to be detected [7]. In addition, the effective field theory describing photon–photon scattering [6] has been widely used to predict possible effects in, for example, extreme magnetized objects, such as neutron stars and magnetars. One effect of a strong magnetic field is to down-shift the frequency of a test photon, the so called photon splitting [8, 9].

In this Letter, we predict that a charged particle moving in an equilibrium radiation gas will emit Cherenkov radiation. The possible importance of this effect is then discussed for the cosmic radiation background. A related, but significantly different, study was presented by Dremin [10]. The dispersion relation for electromagnetic waves

in an isotropic and homogeneous photon gas with refractive index n is $\omega = kc/n$, where $n^2 = 1 + \delta$ and [11, 12]

$$\delta = \frac{2b\alpha\mathscr{E}}{135\pi\epsilon_0 E_S^2} \approx \frac{b\mathscr{E}}{4\times 10^{29}\,\mathrm{J/m^3}}, \tag{1}$$

where b is 8 or 14 depending on the photon polarization, $\alpha \approx 1/137$ is the fine structure constant, ϵ_0 is the permittivity of vacuum, $\mathscr{E}$ is the energy density of the radiation gas, and $E_S \equiv m_e^2c^3/e\hbar \approx 10^{18}\,\mathrm{V/m}$ is the Schwinger field strength. Thus, the refractive index in this case is always larger than one, and a particle may therefore have a speed u exceeding the phase velocity in the medium. The Cherenkov condition $u \geq c/n$ for emission of Cherenkov radiation can thus be satisfied. This condition can also be expressed in terms of the relativistic gamma factor $\gamma = (1-u^2/c^2)^{-1/2}$, namely $\delta\gamma^2 \geq 1$. We will here assume that a particle with charge Ze, satisfying the Cherenkov condition, moves through an equilibrium radiation gas. The energy loss at the frequency ω per unit length of the path of the charged particle is then [13]

$$\frac{dU_\omega}{ds}d\omega = \frac{Z^2\alpha}{c}\frac{(\delta\gamma^2-1)}{(\gamma^2-1)}\hbar\omega\,d\omega, \tag{2}$$

and the number of quanta N emitted per unit length along the particles path is

$$\frac{dN}{ds}d\omega = \frac{Z^2\alpha}{c}\frac{(\delta\gamma^2-1)}{(\gamma^2-1)}\,d\omega. \tag{3}$$

Since δ is normally much less than one, we need a large gamma factor to satisfy the Cherenkov condition. Moreover, the present theory of photon–photon scattering is only valid as long as $\omega \ll \omega_e = m_ec^2/\hbar \approx 8\times 10^{20}\,\mathrm{rad/s}$. The Compton frequency ω_e acts as a cut-off in the integration of the formulas for the energy loss and number of quantas. Subsequently, for $\delta\gamma^2 = 1$, we have

$$U = N\hbar\omega_e, \qquad N = Z^2L\alpha\delta/\lambda_e, \tag{4}$$

respectively. Here L is the distance traveled by the charge, and $\lambda_e = c/\omega_e$ is the Compton wavelength.

At the present time, the cosmic microwave background has an energy density of the order $\mathscr{E} \sim 10^{-15}\,\mathrm{J/m^3}$, i.e. $\delta \sim 10^{-42}$, i.e. the gamma factor has to be $\gamma \geq 10^{21}$ for the Cherenkov condition to be satisfied. Thus Cherenkov radiation is not likely to occur in todays radiation background. In fact, it is well known that the cosmic rays contain non-thermal hadrons, of which some are protons, that can reach gamma factors 10^{11}, but larger values are improbable due to the GZK cut-off [14, 15]. As a comparison, we may consider the situation at the time of matter–radiation decoupling. Since $\mathscr{E}_{\text{emitted}} = \mathscr{E}_{\text{received}}(T/2.7)^4$, where the temperature T is given in Kelvin, we have $\mathscr{E} \sim 10^{-2}\,\mathrm{J/m^3}$ at the time of decoupling ($T \approx 8000\,\mathrm{K}$), implying $\delta \sim 10^{-28}$. Thus, the limiting value on the gamma factor for the Cherenkov condition to be satisfied is $\gamma \geq 10^{14} - 10^{15}$, still out of reach for high energy cosmic rays. However, as we demonstrate below, the situation changes drastically for earlier processes at even higher T. In particular we will focus on the era with $10^9\,\mathrm{K} \leq T \leq 10^{11}\,\mathrm{K}$ when the reguired γ-factors range from $\gamma \sim 10^4$ to $\gamma > 3$.

The effect presented above is then naturally compared with inverse Compton scattering. Setting $Z = 1$, the cross section for this scattering is $\sigma \approx \pi r_e^2 m_e^2/M^2\gamma$, where r_e the classical electron radius and M is the charged particle mass. We thus obtain a collision frequency $\nu = c\mathscr{N}\sigma$, where $\mathscr{N}$ is the number density of the photons. Comparing this frequency with the frequency $\nu_{\mathrm{ch}} = (\gamma Mc)^{-1}dU/dt$, we note that fast particles are mainly scattered due to the Cherenkov effect when $\nu < \nu_{\mathrm{ch}}$, i.e.

$$1 < \frac{\delta}{\alpha\pi(m_e/M)\mathscr{N}\lambda_e^3} = \frac{M}{m_e}\frac{T}{T_{\mathrm{ch}}}. \tag{5}$$

Here T is the temperature of the photon gas, $\mathscr{N} = [30\zeta(3)a/k_B\pi^4]T^3$, $\mathscr{E} = aT^4$, k_B is the Boltzmann constant, $a = \pi^2 k_B^4/15\hbar^3c^3 \approx 7.6 \times 10^{-16}\,\mathrm{J/m^3K^4}$ and $T_{\mathrm{ch}} = 2025\zeta(3)m_ec^2/4\pi^3\alpha b k_B \approx 10^{12}\,\mathrm{K}$. Thus, for a single fast proton to be scattered mainly due to the Cherenkov effect, we need $T > T_{\mathrm{ch}} \times 10^{-3} \sim 10^9\,\mathrm{K}$, well within the limit of validity of the theory for photon–photon scattering. We note that at radiation gas temperatures around $10^{12}\,\mathrm{K}$ the quantum vacuum becomes truly nonlinear, and higher order QED effects should be taken into account.

For the early universe considered above, a moderately relativistic plasma is also present, which means that collective charged particle interactions can play a role. We take these plasma effects into account by introducing the plasma frequency ω_p. The photon dispersion relation then reads $\omega^2 \approx k^2c^2(1-\delta) + \omega_p^2$. Thus the Cherenkov condition is satisfied for charged particles with relativistic factors $\gamma \geq 1/\sqrt{\delta - \omega_p^2/k^2c^2}$. For the temperatures where Cherenkov radiation starts to dominate over inverse Compton scattering, $T \sim 10^9 - 10^{10}\,\mathrm{K}$, we have $\omega_p \sim 10^{15-16}\,\mathrm{rad/s}$, and thus Cherenkov radiation is emitted in a broad band starting in the UV range, $\omega \sim 10^{17}\,\mathrm{rad/s}$, and continuing up to the Compton frequency $\sim 8 \times 10^{20}\,\mathrm{rad/s}$.

The Cherenkov radiation emitted during the era when $T \sim 10^9\,\mathrm{K}$ will be redshifted due to the cosmological expansion. Thus, the present value of the cut-off frequency will be approximately $2 \times 10^{12}\,\mathrm{rad/s}$, i.e., in the short wavelength range of the microwave spectrum. However, we do not expect direct detection of this radiation in the present universe, since the process is expected to be of importance long before the time of radiation decoupling. Still, there are possible important observational implications due to the Cherenkov mechanism presented here. As shown by the inequality (5), the effect will be more pronounced for massive particles with a given gamma factor, and protons are therefore expected to be more constrained than electrons by the QED Cherenkov emission. In particular, (5) puts stronger limits than Compton scattering for suprathermal protons observed today to be relics of the early universe. In fact, it seems rather unlikely, given the inequality (5), that such protons could survive during the $T = 10^9 - 10^{10}\,\mathrm{K}$ era.

Thus, it cannot be excluded that the QED Cherenkov effect in the early universe can be significant even for todays observations.

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